Probabilistic Abstract Argumentation
The constellation approach

Sergio Flesca

DIMES Department
Università della Calabria
Italy

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1. Introduction
   - Probabilistic Abstract Argumentation
   - IND, EX and GEN
2. Computing $\text{P-EXT}^{sem}(S)$ and $\text{P-ACC}^{sem}(a)$
   - Complexity of $\text{P-EXT}^{sem}(S)$ and $\text{P-ACC}^{sem}(a)$ for prAAF of type IND
   - Complexity of $\text{P-EXT}^{sem}(S)$ and $\text{P-ACC}^{sem}(a)$ for prAAF of type EX
   - Complexity of $\text{P-EXT}^{sem}(S)$ and $\text{P-ACC}^{sem}(a)$ for prAAF of type GEN
3. Probabilistic Bipolar argumentation framework
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Argumentation in AI

A general way for representing disputes and debates, with arguments and attack-relationships between arguments.

Abstract Argumentation Framework (AAF) [Dung 1995]: arguments are abstract entities (no attention is paid to their internal structure) that may attack and/or be attacked by other arguments

Example (a simple AAF)

Mary and Marc's defense attorney is reasoning on the trial of a robbery case involving her clients, where Ann is a potential witness.

a: "Mary says that she was at the park at the time of the robbery, and thus denies being involved in the robbery";

b: "Marc says he was at home when the robbery took place, and therefore denies being involved in the robbery";

c: "Ann says that she certainly saw Mary near the bank just before the robbery, and possibly saw Marc there too".

Several ways of modeling uncertainty in AAFs: weights, preferences, probabilities.

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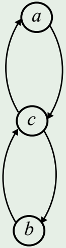

Several ways of modeling uncertainty in AAFs: weights, preferences, probabilities.

Probabilistic Abstract Argumentation Framework: Epistemic and Constellation approaches

Two different way of modelling uncertainty in abstract argumentation using a probabilistic approach.

- *Epistemic* [Hunter 2013, Hunter and Thimm 2014]: uncertainty about the fact that an argument is justifiable by an agent, i. e., that both the premises of the argument and the derivation of the claim of the argument from its premises are valid.
- ***Constellation*** [Dung and Thang 2010, Li et Al. 2011]: uncertainty about the fact that an argument/attack is in the framework

Probabilistic Abstract Argumentation Framework (prAAF)

- [Dung and Thang 2010] proposed EX (shorthand for "*extensive*"): uncertainty is taken into account by extensively specifying a probability distribution function (pdf) over the possible scenarios.

Example

According to EX, suppose that the lawyer thinks that only the following 4 scenarios are possible:
S_1: "Ann does not testify"; $\alpha_1 = \langle\{a, b\}, \emptyset\rangle$
S_2: "Ann testifies, and the jury will deem that argument *c* undermines Mary and Marc's arguments *a*, *b*, and vice versa"; $\alpha_2 = \langle\{a, b, c\}, \{\delta_{ac}, \delta_{ca}, \delta_{bc}, \delta_{cb}\}\rangle$
S_3: "Ann testifies, and the jury will deem that her argument *c* undermines Mary and Marc's arguments *a*, *b*, while, owing to the bad reputations of Mary and Marc, *a* and *b* will be not perceived as strong enough to undermine argument *c*";
$\alpha_3 = \langle\{a, b, c\}, \{\delta_{ca}, \delta_{cb}\}\rangle$
S_4: "Ann testifies, and the jury will deem that her argument *c* undermines Mary's argument *a* but not Marc's argument *b*, as Ann is uncertain about Marc's presence. Vice versa, *a* and *b* will be not perceived as strong enough to undermine *c*".
$\alpha_4 = \langle\{a, b, c\}, \{\delta_{ca}\}\rangle$

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Example

According to EX, suppose that the lawyer thinks that only the following 4 scenarios are possible:
S_1: "Ann does not testify"; $\alpha_1 = \langle\{a,b\},\emptyset\rangle$ $P(\alpha_1) = 0.1$
S_2: "Ann testifies, and the jury will deem that argument c undermines Mary and Marc's arguments a, b, and vice versa"; $\alpha_2 = \langle\{a,b,c\},\{\delta_{ac},\delta_{ca},\delta_{bc},\delta_{cb}\}\rangle$ $P(\alpha_2) = 0.3$
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Probabilistic Abstract Argumentation Framework (prAAF)

- [Li et Al. 2011] proposed IND (shorthand for "*independence*"): each argument/defeat can be associated with a probability (and arguments and defeats are viewed as *independent* events);

Example

The lawyer assigns:

- $P(c) = 0.9$ (meaning that there is 10% probability that Mary will not testify),
- $P(a) = P(b) = 1$ (meaning that Mary and Marc will certainly testify),
- $P(\delta_{ca}) = 1$ (meaning that she/he is certain that the jury will consider Ann's argument as a solid rebuttal of Mary's argument),
- $P(\delta_{cb}) = 0.8$ and $P(\delta_{ac}) = P(\delta_{bc}) = 0.4$.

Possible scenarios:

- $\alpha_1, \ldots \alpha_4$ of the previous example,
- $\alpha_5 = \langle\{a, b, c\}, \{\delta_{ac}, \delta_{ca}\}\rangle$,
- $\alpha_6 = \langle\{a, b, c\}, \{\delta_{ca}, \delta_{bc}\}\rangle$
- $\alpha_7 = \langle\{a, b, c\}, \{\delta_{ca}, \delta_{cb}, \delta_{ac}\}\rangle$,
- $\alpha_8 = \langle\{a, b, c\}, \{\delta_{ca}, \delta_{cb}, \delta_{bc}\}\rangle$,
- $\alpha_9 = \langle\{a, b, c\}, \{\delta_{ca}, \delta_{ac}, \delta_{bc}\}\rangle$.

$P(\alpha_5) = P(a) \times P(b) \times P(c) \times P(\delta_{ac}) \times P(\delta_{ca}) \times (1 - P(\delta_{bc})) = 0.17$

Semantics in Abstract Argumentation

Semantics for AAFs, i.e., identifying "reasonable" sets of arguments, called *extensions*: *admissible*, *stable*, *preferred*, *complete*, *grounded*, *semi-stable*, *ideal-set*, *ideal*.

– a set S of arguments is an *admissible* extension if it is "*conflict-free*" (i.e., there is no defeat between arguments in S), and every argument attacking arguments in S is counterattacked by an argument in S

Classical problems in Abstract Argumentation

- $\textsc{Ext}^{sem}(S)$: verifying whether S is an extension according to a semantics *sem*,
- $\textsc{CA}^{sem}$: deciding whether argument a is *acceptable* (i.e., it belongs to some extension according to *sem*)
- $\textsc{P-Ext}^{sem}(S)$: what is the probability that S is an extension according to a semantics *sem*?
- $\textsc{P-Acc}^{sem}(a)$: what is the probability that argument a is *acceptable* (i.e., it belongs to some extension according to *sem*)?

P-$\text{EXT}^{sem}(S)$

- The probability $Pr^{sem}(S)$ that a set S of arguments is reasonable according to a given semantics *sem* is defined as *the sum of the probabilities of the possible worlds w for which S is reasonable according to sem*

Example (probability that $\{a, b\}$ is an admissible set)

In our example first example, the possible worlds for which $\{a, c\}$ is admissible are:
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P-ACC$^{sem}(a)$

- The probability $Pr^{sem}_{acc}(a)$ that an argument a is (credulously) acceptable according to a given semantics *sem* is defined as *the sum of the probabilities of the possible worlds w for which a is acceptable according to sem*

Example (probability that b is acceptable according to the admissible semantics)

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Hence $Pr^{sem}_{acc}(b) = 0.7$

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IND VS EX

How to choose between EX and IND!

- EX: expressive but not compact.
  - It requires to provide an estimation of the probability of every possible scenario. How to do this? How much time it will take?
- IND: compact but with limited expressiveness
  - It only requires to provide an estimation of the arguments/defeats' probabilities (marginal probabilities are easier to be estimated). It assumes that there is no correlation between arguments (e.g. it is not possible to specify that two arguments occur together)

GEN

- generalizes EX, since it also enables an “extensive” definition of the pdf over the possible AAFs;
- generalizes IND, since it also allows us to impose independence between arguments/defeats;
- in order to encode a pdf over the possible AAFs, it exploits the representation model of *world-set descriptors* and *world-set sets*, that is a succinct and complete model for representing possible worlds and probabilities over them [Antova et al 2008, Koch and Olteanu 2008]
- allows the specification of correlations: for instance, co-existence and XOR

Restrictions:

In order to study if and how the complexity varies when imposing a restriction, two restricted forms of GEN, named IND-A (independence over arguments) and IND-D (independence over defeats) were introduced

World set descriptors (*wsds*) and world-set sets (*ws-sets*)

Possible worlds

Let V be a finite set of variables, where, $\forall x \in V$, the domain of x is finite and is denoted as Dom_x.
A valuation over V is a set of assignments (each of which is denoted as $x \mapsto i$, with $x \in V$ and $i \in Dom_x$) containing at most one assignment for each variable in V.
A possible world over V is a total *valuation* over V.

Example

For instance, over the set of binary variables $V = \{x, y\}$, the following four possible worlds are defined:

- $w_1 = \{x \mapsto 0, y \mapsto 0\}$;
- $w_2 = \{x \mapsto 0, y \mapsto 1\}$;
- $w_3 = \{x \mapsto 1, y \mapsto 0\}$;
- $w_4 = \{x \mapsto 1, y \mapsto 1\}$.

wsds and *ws-sets*

As shown in [Koch and Olteanu 2008], assuming that the variables in V are independent random variables allows us to implicitly encode any pdf over the set of possible worlds over V.

The pdfs of the variables in V are encoded in a *world table W* containing, for each $x \in V$ and $i \in Dom_x$, a triple (x, i, p), where p is the probability $P(\{x \mapsto i\})$ that the value of x is i.

The probability of a possible world w is the product of the probabilities of the assignments defining w. For instance, the probability of the possible world $w_1 = \{x \mapsto 0, y \mapsto 0\}$ is $P(\{x \rightarrow 0\}) \cdot P(\{y \rightarrow 0\})$.

wsds and *ws-sets*

A *wsd* d over V is a valuation over V, and it describes the set $\omega(d)$ of the possible worlds encoded by the total valuations extending d.

Example

For instance, in the above-discussed case where $V = \{x, y\}$ and x, y are binary variables, the wsd $d = \{y \mapsto 0\}$ describes the set of possible worlds $\{w_1, w_3\}$, as $w_1 = \{x \mapsto 0, y \mapsto 0\}$ and $w_3 = \{x \mapsto 1, y \mapsto 0\}$ are the only two total valuations over V extending d.

- Owing to the independence of the variables in V, the cumulative probability of the possible worlds in $\omega(d)$ is $P(d) = \prod_{\{x \mapsto i\} \subseteq d} P(\{x \mapsto i\})$.
- A ws-set S is a set of ws-descriptors and represents the set of possible worlds resulting from the union of the sets of possible worlds represented by the ws-descriptors in S, i.e. $\omega(S) = \cup_{d \in S} \omega(d)$.
- The probability $P(S)$ of a ws-set S is the sum of the probabilities of the possible worlds in $\omega(S)$.

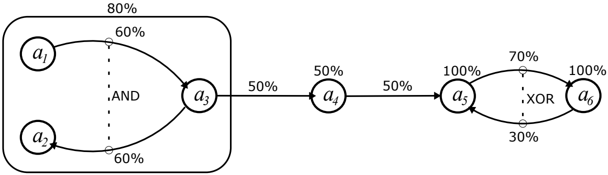
prAAFs of the form GEN

Definition::

A prAAF of the form GEN is a tuple $\mathcal{F} = \langle A, D, W, \lambda\rangle$, where A is a set of arguments, D a set of defeats, W a world table, and $\lambda : A \cup D \to WS(W)$ is a function assigning every argument and defeat with a ws-set over W.

A possible world $w \in \omega(W)$ *supports* the possible AAF $\alpha = \langle A', D'\rangle$ (denoted as $w \models \alpha$) if every argument/defeat $\sigma \in A' \cup D'$ is such that $w \in \omega(\lambda(\sigma))$, and there are no argument $a \in A \setminus A'$ such that $w \in \omega(\lambda(a))$ and no defeat $\delta \in (A' \times A') \setminus D'$ such that $w \in \omega(\lambda(\delta))$.

prAAFs of the form GEN: Example



Var	*value*	p
x_{123}	0	0.2
x_{123}	1	0.8
x_4	0	0.5
x_4	1	0.5
x_5	1	1
x_6	1	1

Term	$\lambda(\mathit{Term})$
a_1	$\{\{x_{123} \to 1\}\}$
a_2	$\{\{x_{123} \to 1\}\}$
a_3	$\{\{x_{123} \to 1\}\}$
a_4	$\{\{x_4 \to 1\}\}$
a_5	$\{\{x_5 \to 1\}\}$
a_6	$\{\{x_6 \to 1\}\}$

Var	*value*	p
$y_{13,32}$	0	0.4
$y_{13,32}$	1	0.6
y_{34}	0	0.5
y_{34}	1	0.5
y_{45}	0	0.5
y_{45}	1	0.5
$y_{56,65}$	0	0.7
$y_{56,65}$	1	0.3

Term	$\lambda(Term)$
δ_{13}	$\{\{y_{13,32} \to 1\}\}$
δ_{32}	$\{\{y_{13,32} \to 1\}\}$
δ_{34}	$\{\{y_{34} \to 1\}\}$
δ_{45}	$\{\{y_{45} \to 1\}\}$
δ_{56}	$\{\{y_{56,65} \to 0\}\}$
δ_{65}	$\{\{y_{56,65} \to 1\}\}$

Restricted forms of GEN

Definition:: *Boolean prAAF* (BOOL).

prAAF $\mathcal{F} = \langle A, D, W, \lambda \rangle$ of the form GEN is said to be *boolean* (of the form BOOL) if, for each $x \in Var(W)$, Dom_x is the boolean domain.

- Intuitively, boolean prAAFs allow the occurrences of arguments and defeats within a dispute to be defined in terms of boolean formulas over a set of "elementary" independent probabilistic events
- Any "boolean" ws-set $\{wsd_1, \ldots, wsd_n\}$ encodes the DNF formula $c(wsd_1) \vee \cdots \vee c(wsd_n)$, where, in turn, for each $wsd_i = \{x_1 \mapsto true, \ldots, x_k \mapsto true, x_{k+1} \mapsto false, \ldots, x_m \mapsto false\}$, the term $c(wsd_i)$ is the conjunction $x_1 \wedge \cdots \wedge x_k \wedge \neg x_{k+1} \wedge \cdots \wedge \neg x_m$.

Restricted forms of GEN

Definition:: *Monadic prAAF* (MON).

prAAF $\mathcal{F} = \langle A, D, W, \lambda \rangle$ is said to be *monadic* (of the form MON) iff it is boolean and, for each $\sigma \in A \cup D$, $\lambda(\sigma) = \{\{x \mapsto v\}\}$, where $x \in Var(W)$ and $v \in Dom_x$.

- Intuitively, monadic prAAFs allow us to express co-existence of arguments/defeats and mutual exclusiveness between pairs of arguments and defeats in terms of *XOR constraints*.
- An XOR constraint between two arguments/defeats σ_1 and σ_2 states that any possible AAF contains either σ_1 or σ_2, but not both.
- The co-existence of a set of arguments/defeats $\{\sigma_1, \ldots, \sigma_k\}$ is imposed by assigning the same ws-set to each of them, i.e., $\lambda(\sigma_1) = \cdots = \lambda(\sigma_k)$, where $\lambda(\sigma_1)$ is of the form $\{\{x\}\}$ or $\{\{\neg x\}\}$.
- An XOR constraint over a pair σ_1, σ_2 of arguments/defeats is imposed by using the negation of the literal describing σ_1 as descriptor for σ_2.

Restricted forms of GEN

Definition:: ***Monadic prAAF with independent defeats*** **(IND-D).**

monadic prAAF $\mathcal{F} = \langle A, D, W, \lambda \rangle$ is said to be *monadic with independent defeats* (of the form IND-D) iff, for each $\delta \in D$, given that $\lambda(\delta) = \{\{x \mapsto v\}\}$, there is no argument/defeat $\sigma \in A \cup D$ such that $\lambda(\sigma) = \{\{x \mapsto v'\}\}$, with $v, v' \in \{$*true, false*$\}$.

- A prAAFs of the form IND-D still allow us to impose the co-existence of arguments and XOR constraints over pairs of arguments. On the other hand, in this form of prAAF, defeats are modeled as conditionally independent from one another, given the occurrence of the arguments over which they are defined.
- This case has been considered in some well-known frameworks in the literature, such as the framework addressed in [Dondio 2014], where defeats (but not the arguments) are assumed to be certain (obviously, “certainty” is a particular case of independence).

Restricted forms of GEN

Definition:: *Monadic prAAF with independent arguments* (IND-A).

monadic prAAF $\mathcal{F} = \langle A, D, W, \lambda \rangle$ is said to be *monadic with independent arguments* (or, equivalently, of the form IND-A) iff, for each $a \in A$, given that $\lambda(a) = \{\{x \mapsto v\}\}$, there is no argument/defeat $\sigma \in A \cup D$ such that $\lambda(\sigma) = \{\{x \mapsto v'\}\}$, with $v, v' \in \{$*true, false*$\}$.

- prAAFs of type IND-A allow us to impose the co-existence of defeats and XOR constraints over pairs of defeats, while modeling the occurrences of different arguments within the dispute as independent events.
- The case where arguments are independent while defeats can be correlated is at the basis of the study in [Hunter 2014], where a framework for probabilistically modeling attacks (while arguments are certain) has been introduced.

Computing P-$\text{EXT}^{sem}(S)$ and P-$\text{ACC}^{sem}(a)$

How to compute P-$\text{EXT}^{sem}(S)$ and P-$\text{ACC}^{sem}(a)$?

- The number of possible worlds may be huge: is it more reasonable to estimate them?
- This was proposed in [Li et Al. 2011]!
- However, maybe in same cases we can provide the exact answers in reasonable time, when?
  - We need to characterize the complexity of P-$\text{EXT}^{sem}(S)$ and P-$\text{ACC}^{sem}(a)$
  - P-$\text{EXT}^{sem}(S)$ and P-$\text{ACC}^{sem}(a)$ are both function problems

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A glance at complexity classes: $\#P$, $FP^{\#P}$ and $FP^{||NP}$

- $\#P$ is the complexity class of the functions f such that f counts the number of accepting paths of a nondeterministic polynomial-time Turing machine
- Although P-EXT$^{sem}(S)$ and P-ACC$^{sem}(a)$ are closely related to $\#P$, strictly speaking, it cannot belong to it, since the outputs of our problem are not integers
- $FP^{\#P}$ is the class of functions that are computable by a polynomial-time Turing machine with a $\#P$ oracle
  - a function is $FP^{\#P}$-hard iff it is $\#P$-hard, and thus to prove that a problem is $FP^{\#P}$-hard it suffices to reduce a $\#P$-hard problem to it
  - For each complexity class $\#\mathcal{C} \in \#PH$, it holds that $FP^{\#P} = FP^{\#\mathcal{C}}$ (since $\#PH \subseteq FP^{\#P[1]}$ under polynomial time 1-Turing reductions [Toda and Watanabe 1992])
- $FP^{||NP}$ (resp., $FP^{||\Sigma_p^2}$) is the class of functions computable by a polynomial-time Turing machine with access to an NP (Σ_p^2) oracle, whose calls are non-adaptive

Comlexity results for P-$\textsc{Ext}^{sem}(S)$

Complexity of $\textsc{Ext}^{sem}(S)$ and P-$\textsc{Ext}^{sem}(S)$ for different forms of prAAFs

sem	$\textsc{Ext}^{sem}(S)$	P-$\textsc{Ext}^{sem}(S)$			
		IND	EX	GEN, BOOL, MON, IND-A	IND-D
admissible	P	FP	FP	$FP^{\#P}$-c	FP
stable	P	FP	FP	$FP^{\#P}$-c	FP
complete	P	$FP^{\#P}$-c	FP	$FP^{\#P}$-c	$FP^{\#P}$-c
grounded	P	$FP^{\#P}$-c	FP	$FP^{\#P}$-c	$FP^{\#P}$-c
semi-stable	$coNP$-c	$FP^{\#P}$-c	$FP^{\|\|NP}$-c	$FP^{\#P}$-c	$FP^{\#P}$-c
preferred	$coNP$-c	$FP^{\#P}$-c	$FP^{\|\|NP}$-c	$FP^{\#P}$-c	$FP^{\#P}$-c
ideal-set	$coNP$-c	$FP^{\#P}$-c	$FP^{\|\|NP}$-c	$FP^{\#P}$-c	$FP^{\#P}$-c
ideal	in Θ_2^p, $coNP$-h	$FP^{\#P}$-c	$FP^{\|\|NP}$-c	$FP^{\#P}$-c	$FP^{\#P}$-c

Comlexity results for P-ACC$^{sem}(a)$

Complexity of CA^{sem} and P-ACC$^{sem}(a)$ for different forms of prAAFs

sem	CA^{sem}	P-ACC$^{sem}(a)$ IND	P-ACC$^{sem}(a)$ EX	P-ACC$^{sem}(a)$ GEN, BOOL, IND-D MON, IND-A
admissible	NP-c	$FP^{\#P}$-c	$FP^{\|\|NP}$-c	$FP^{\#P}$-c
stable	NP-c		$FP^{\|\|NP}$-c	
complete	NP-c		$FP^{\|\|NP}$-c	
grounded	P		FP	
semi-stable	Σ_p^2-c		in $FP^{\|\|\Sigma_p^2}$, $FP^{\|\|NP}$-h	
preferred	NP-c		$FP^{\|\|NP}$-c	
ideal-set	in Θ_2^p, $coNP$-h		$FP^{\|\|NP}$-c	
ideal	in Θ_2^p, $coNP$-h		$FP^{\|\|NP}$-c	

What is the complexity of P-$\text{EXT}^{sem}(S)$ for prAAF of type IND?

- Computing $Pr^{sem}(S)$ by directly applying the definition would require exponential time (it relies on summing the probabilities of an exponential number of possible worlds)
- P-$\text{EXT}^{sem}(S)$ can be solved in time $O(|S| \cdot |A|)$ for the *admissible* and *stable* semantics
- P-$\text{EXT}^{sem}(S)$ is $FP^{\#P}$-complete for the *complete, grounded, preferred,ideal set,* and *ideal* semantics

Main idea for designing a PTIME algorithm

- Express the fact that a set S of arguments is admissible [resp., stable] as a probabilistic event $E_{ad}(S)$ [resp., $E_{st}(S)$]
- $Pr^{admissible}(S) = Pr(E_{ad}(S))$ [resp., $Pr^{stable}(S) = Pr(E_{st}(S))$]
- the tractability of PROB$^{admissible}(S)$ [resp. PROB$^{stable}(S)$] follows from the fact that $Pr^{admissible}(S)$ [resp., $Pr^{stable}(S)$)] results in a polynomial-size expression involving only the probabilities of the arguments and the defeats
- this does not hold for the other semantics (*complete, grounded, preferred, and ideal*)

Admissible semantics - probabilistic event

- $E_{ad}(S) = e_1(S) \wedge e_2(S) \wedge e_3(S)$
- $e_1(S)$ is the event that all of the arguments in S occur
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- $e_3(S)$ is the event that, given that $e_1(S)$ holds, for all the arguments d outside S, one of the following events holds:
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Lemma

$Pr^{admissible}(S) = Pr(E_{ad}(S)) = Pr(e_1(S)) \cdot Pr(e_2(S)) \cdot Pr(e_3(S))$

The probabilities of e_1, e_2, and e_3 are as follows (next slides)

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Probability that a set is admissible (1/2)

- $E_{ad}(S) = e_1(S) \land e_2(S) \land e_3(S)$
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Example (probability that $\{a,c\}$ is admissible (to be continued))

$$Pr^{admissible}(\{a,c\}) = \underbrace{P_A(a) \cdot P_A(c)}_{Pr(e_1(\{a,c\}))} \cdot \underbrace{1}_{Pr(e_2(\{a,c\}))} \cdot Pr(e_3(S))$$

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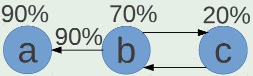

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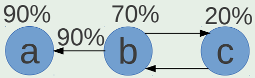
Tractability of admissible semantics

Example (probability that $\{a, c\}$ is admissible (continued))



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Theorem

$\textsc{Prob}^{admissible}(S)$ *can be solved in time* $O(|S|\cdot|A|)$.

Tractability of admissible semantics

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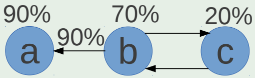

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Stable semantics

- probabilistic event that S is stable: $E_{st}(S) = e_1(S) \wedge e_2(S) \wedge e_3'(S)$
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PROB$^{stable}(S)$ *can be solved in time* $O(|S| \cdot |A|)$.

$FP^{\#P}$-complete cases

Theorem

P-EXT$^{sem}(S)$ *is* $FP^{\#P}$*-complete for sem in* $\{$*complete, grounded, semi-stable, preferred ideal-set, ideal* $\}$

- For all the semantics but the ideal-set one:
  - reduction from the $\#P$-hard problem $\#PP2DNF$ (*Partitioned Positive 2DNF*)
  - $\#PP2DNF$ is the problem of counting the number of satisfying assignments of a DNF formula $\phi = C_1 \vee C_2 \vee \cdots \vee C_k$ whose propositional variables are positive and can be partitioned into two sets $X = \{x_1, \ldots, x_n\}$ and $Y = \{y_1, \ldots, y_m\}$, and each clause C_i has the form $x_j \wedge y_\ell$, with $x_j \in X$ and $y_\ell \in Y$
- For ideal-set semantics:
  - reduction from $\#P2CNF$ (the problem of counting the number of satisfying assignments of a positive 2CNF formula)

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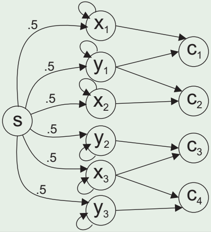
$FP^{\#P}$ hardness for P-EXT$^{\text{complete}}(S)$ (1/2)

Given ϕ, consider the PrAF $\mathcal{F}_\phi = \langle A, P_A, D, P_D \rangle$ such that

- A contains an argument for each propositional variable in ϕ, an argument c_ℓ for each clause C_ℓ of ϕ, and an argument s;
- D contains the defeats (x_i, c_ℓ) and (y_j, c_ℓ) for each clause $C_\ell = x_i \wedge y_j$ of ϕ, and the defeats (s, x_i) and (x_i, x_i) (resp., (s, y_j) and (y_j, y_j)) for each variable x_i (resp., y_j) of ϕ;
- P_A assigns probability 1 to all the arguments in A; P_D assigns probability 1 to all the defeats in D except the defeats (s, x_i) and (s, y_j), which are assigned .5

Example

$$\phi' = (x_1 \wedge y_1) \vee (x_2 \wedge y_1) \vee (x_3 \wedge y_2) \vee (x_3 \wedge y_3)$$

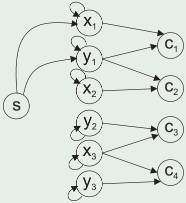


$FP^{\#P}$ hardness for PROB$^{complete}(S)$ (2/2)

- there is a bijection $b : \mathcal{T} \rightarrow pw(\mathcal{F})$ between the set $\mathcal{T}$ of truth assignments of ϕ and the set $pw(\mathcal{F})$ of possible worlds
- given a truth assignment τ for the variables of ϕ, ϕ evaluates to *true* under τ iff $S = \{s\}$ is not a complete extension in the world $w = b(\tau)$

Example

- $\tau' = x_1/1, x_2/0, x_3/0, y_1/1, y_2/0, y_3/0$ for ϕ' corresponds to the world $w_{\tau'}$
- τ' makes ϕ' true and $\{s\}$ is not a complete extension in $w_{\tau'}$, since s defeats both x_1 and y_1 which makes c_1 acceptable w.r.t $\{s\}$



- it can be shown that the number of satisfying assignments of ϕ is equal to $2^{n+m} \cdot (1 - Pr_{\mathcal{F}}^{complete}(\{s\}))$

$FP^{\#P}$ membership for $\text{PROB}^{complete}(S)$ (1/2)

$Pr_{\mathcal{F}}^{complete}(S)$ can be computed by a polynomial time algorithm $\mathcal{A}$ with access to a $\#P$ oracle

- $Pr_{\mathcal{F}}^{complete}(S)$ can be expressed as a rational number whose denominator d is the product of the denominators of the probabilities of arguments in A and defeats in D
- Algorithm $\mathcal{A}$ first computes d in polynomial time w.r.t. the size of $\mathcal{F}$, then calls a $\#P$ oracle to determine the numerator of $Pr_{\mathcal{F}}^{complete}(S)$
- algorithm $\mathcal{A}$ returns both n and d.

$FP^{\#P}$ membership for $\textsc{Prob}^{complete}(S)$ (2/2)

- The oracle counts the number of accepting paths of a nondeterministic polynomial-time Turing machine M such that:
  - (i) M nondeterministically guesses a subset of arguments in A and defeats in D so that each leaf of the resulting computation tree is a possible world $w \in pw(\mathcal{F})$
  - (ii) At each leaf, let w be the guessed world, and $I(w)$ its probability, the computation tree is then split again $d \cdot I(w)$ times to reflect the probability of the guessed world (for each $w \in pw(\mathcal{F})$, $I(w)$ is a rational number whose denominator is d, and $I(w)$ can be computed in polynomial time w.r.t. the size of $\mathcal{F}$)
  - (iii) Finally, M checks in polynomial time if S is a complete set of arguments in the world w
- the number of accepting paths of M is $d \cdot Pr_{\mathcal{F}}^{complete}(S)$, that is the numerator n of $Pr_{\mathcal{F}}^{complete}(S)$
- algorithm $\mathcal{A}$ returns both n and d.

$FP^{\#P}$ membership for P-$\textsc{Ext}^{\text{preferred}}(S)$

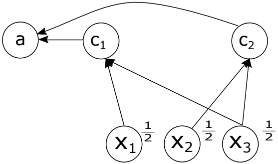
- $Pr_{\mathcal{F}}^{preferred}(S)$ can be computed by algorithm A:
- A first computes (in polynomial time) the denominator d of $Pr_{\mathcal{F}}^{preferred}(S)$
- Then, A invokes a $\#NP$ oracle that counts the number of accepting path of a non-deterministic Turing machine M such that:
  - (i) M nondeterministically guesses a subset of arguments in A and defeats in D so that each leaf of the resulting computation tree is a possible world $w \in pw(\mathcal{F})$
  - (ii) At each leaf, let w be the guessed world, and $I(w)$ its probability, the computation tree is then split again $d \cdot I(w)$ times to reflect the probability of the guessed world (for each $w \in pw(\mathcal{F})$, $I(w)$ is a rational number whose denominator is d, and $I(w)$ can be computed in polynomial time w.r.t. the size of $\mathcal{F}$)
  - (iii) Finally, M invokes an NP oracle that checks whether S is a preferred set of arguments in the world w
- Remember that $FP^{\#P} = FP^{\#NP}$ [Toda and Watanabe 1992].

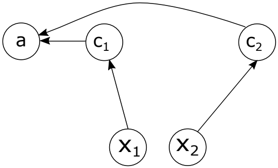
P-$\textsc{Acc}^{sem}_{\text{IND}}(a)$

Theorem

For sem $\in\{\mathtt{ad}, \mathtt{st}, \mathtt{co}, \mathtt{gr}, \mathtt{sst}, \mathtt{pr}, \mathtt{ids}, \mathtt{ide}\}$, *it holds that* P-$\textsc{Acc}^{sem}(a)$ *is* $FP^{\#P}$*-complete.*

- *Membership*: similar to P-$\textsc{Ext}^{sem}_{\text{IND}}(S)$.
- *Hardness*: reduction from the $\#P$-hard problem $\#P2CNF$, that is the problem of counting the number of satisfying assignments of a *CNF* formula where each clause consists of exactly 2 positive literals, to P-$\textsc{Acc}^{sem}(a)$.





Graphical representation of the PrAAF $\mathcal{F}(\phi)$, where $\phi = (X_1 \vee X_3) \wedge (X_2 \vee X_3)$ and the possible world $w(\gamma, \mathcal{F}(\phi))$, where γ is the truth assignment for X_1, X_2 and X_3 such that $\gamma(X_1) = \mathtt{true}$, $\gamma(X_2) = \mathtt{true}$ and $\gamma(X_3) = \mathtt{false}$

Tractable cases for P-$\textsc{Ext}^{sem}_{\mathrm{EX}}(S)$

Theorem

P-$\textsc{Ext}^{sem}_{\mathrm{EX}}(S)$ *is in FP for sem* $\in \{\mathtt{ad}, \mathtt{st}, \mathtt{gr}, \mathtt{co}\}$.

- for every *sem* $\in \{\mathtt{ad}, \mathtt{st}, \mathtt{gr}, \mathtt{co}\}$, deciding whether S is an extension in a deterministic AAF (i.e., solving $\textsc{Ext}^{sem}(S)$) is in PTIME;
- the number of possible AAFs over which this check must be performed is linear in the input.

Hard cases for P-$\text{EXT}_{\text{EX}}^{sem}(S)$

Theorem

P-$\text{EXT}_{\text{EX}}^{sem}(S)$ *is* $FP^{||NP}$*-complete for sem* $\in \{\mathtt{pr}, \mathtt{ids}, \mathtt{ide}, \mathtt{sst}\}$.

Membership in $FP^{||NP}$.

- For the semantics $\mathtt{sst}$, $\mathtt{pr}$ and $\mathtt{ids}$ (for which $\text{EXT}^{sem}(S)$ is coNP-complete), the membership in $FP^{||NP}$ follows from the fact that P-$\text{EXT}_{\text{EX}}^{sem}(S)$ can be solved by performing as many parallel invocations to NP oracles (each solving an instance of $\text{EXT}^{sem}(S)$ over a possible AAF with non-zero probability) as the number of possible AAFs encoded in the prAAF.
- For the semantics $\mathtt{ide}$, the membership to $FP^{||NP}$ still holds, since a polynomial time Turing machine with parallel invocations to Θ_2^p oracles can be easily converted into a polynomial time Turing machine with parallel invocations to NP oracles ($\Theta_2^p = P^{||NP}$).

Hard cases for P-$\textsc{Ext}^{sem}_{\textsc{ex}}(S)$

Theorem

P-$\textsc{Ext}^{sem}_{\textsc{ex}}(S)$ *is $FP^{||NP}$-complete for sem* $\in \{\mathtt{pr}, \mathtt{ids}, \mathtt{ide}, \mathtt{sst}\}$.

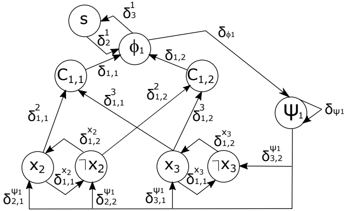
Hardness for $FP^{||NP}$.

- Reduction from the $FP^{||NP}$-hard problem *sup*(ϕ), that is the problem of computing the *supremum* of the satisfying assignments for a 3CNF Boolean formula $\phi(x_1, \ldots, x_n)$.
- The supremum *sup*(ϕ) is the assignment where, for each $i \in [1..n]$, the variable x_i is assigned with *true* iff there exists a satisfying assignment of ϕ wherein $x_i = true$.

Hard cases for P-Ext$^{sem}_{\text{EX}}(S)$

Reduction for `ide`.

- Let $\phi = C_1 \wedge C_2 \wedge \cdots \wedge C_k$ be the 3-CNF boolean formula in the instance of supremum, over the set $X = \{x_1, \ldots, x_n\}$ of variables.
- Build n formulas $\phi_1, \ldots, \phi_n$ from ϕ, where each ϕ_i has the form $\phi_i = C_{i,1} \wedge C_{i,2} \wedge \cdots \wedge C_{i,k_i}$ and is obtained from ϕ by assigning $x_i = true$.
- Define the prAAF $\mathcal{F}_\phi = \langle A, D, \vec{\alpha}, \vec{P} \rangle$ of the form EX defining a possible AAF $\alpha_i = \langle A_i, D_i \rangle$ for each formula ϕ_i, whose probability is $P_i = 2^{i-1}/2^n$.



AAF α_1, where $\phi_1 = (x_2 \vee x_3) \wedge (\neg x_2 \vee x_3)$ is obtained from the 3-CNF formula $\phi = (\neg x_1 \vee x_2 \vee x_3) \wedge (\neg x_1 \vee \neg x_2 \vee x_3) \wedge (x_1 \vee x_4 \vee x_5)$ by assigning $x_1 = true$

$\{s\}$ is not an ideal-set extension (and, thus, it is not an ideal extension) in the AAF $\langle A_i, D_i \rangle$ iff there exists a truth assignment t for $x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n$ making ϕ_i evaluate to true.

Complexity of P-$\text{Acc}^{sem}_{\text{EX}}(a)$

Theorem

P-$\text{Acc}^{sem}_{\text{EX}}(a)$ *is in FP for sem* $= \mathtt{gr}$.

Theorem

P-$\text{Acc}^{sem}_{\text{EX}}(a)$ *is in* $FP^{||NP}$ *for sem* $\in \{\mathtt{ad}, \mathtt{st}, \mathtt{co}, \mathtt{pr}, \mathtt{ids}, \mathtt{ide}\}$*, and in* $FP^{||\Sigma_p^2}$ *for sem* $= \mathtt{sst}$.

Theorem

P-$\text{Acc}^{sem}_{\text{EX}}(a)$ *is* $FP^{||NP}$*-hard for sem* $\in \{\mathtt{ad}, \mathtt{st}, \mathtt{co}, \mathtt{pr}, \mathtt{ids}, \mathtt{ide}, \mathtt{sst}\}$

Upper Bound of Complexity for prAAF of type $\textsc{gen}$

Theorem

For any $sem \in SEM$, P-$\textsc{Ext}^{sem}_{\textsc{gen}}(S)$ and P-$\textsc{Acc}^{sem}_{\textsc{gen}}(a)$ are in $FP^{\#P}$.

The proof is similar to the proof of membership in $FP^{\#P}$ for prAAF of type $\textsc{ind}$.

Hard Cases

Theorem

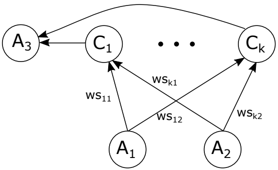
For any sem $\in \{\mathtt{ad}, \mathtt{st}, \mathtt{co}, \mathtt{gr}, \mathtt{pr}, \mathtt{ids}, \mathtt{ide}, \mathtt{sst}\}$, P-EXT$^{sem}_{\text{IND-A}}(S)$ *is* $FP^{\#P}$*-hard.*

- $FP^{\#P}$-hardness for *sem* $\in \{\mathtt{co}, \mathtt{gr}, \mathtt{pr}, \mathtt{ids}, \mathtt{ide}, \mathtt{sst}\}$ is implied by the fact that P-EXT$^{sem}_{\text{IND}}(S)$ is $FP^{\#P}$-complete and IND can be seen as a further restriction of IND-A.
- For *sem* $\in \{\mathtt{ad}, \mathtt{st}\}$ there is a reduction to P-EXT$^{sem}(S)$from the $\#P$-hard problem $\#P2CNF$, that is, the problem of counting the number of satisfying assignments of a *CNF* formula where each clause consists of exactly 2 positive literals.

reduction from $\#P2CNF$ to P-Ext$^{sem}(S)$

Let $\phi = C_1 \wedge C_2 \wedge \ldots C_k$ be a *P2CNF*, where $X = \{x_1, \ldots, x_n\}$ is the set of its propositional variables. We define the prAAF $\mathcal{F}_\phi = \langle A, D, W, \lambda \rangle$ of the form IND-A where:

- The set A consists of: (i) three arguments A_j with $j \in [1..3]$; and (ii) an argument C_i for each clause C_i appearing in ϕ;
- The relation D contains, for each clause C_i (with $i \in [1..k]$), a defeat $\delta^1_{C_i} = (A_1, C_i)$, a defeat $\delta^2_{C_i} = (A_2, C_i)$, and a defeat $\delta^3_{C_i} = (C_i, A_3)$.
- The world table W contains a triple $\langle x, true, 1 \rangle$, and, for each $i \in [1..n]$, the two triples $\langle x_i, true, \frac{1}{2} \rangle$ and $\langle x_i, false, \frac{1}{2} \rangle$;
- Function λ is defined as follows: *i*) for each $a \in A$, $\lambda(a) = \{x \mapsto true\}$; and *ii*) for each $i \in [1..k]$, $\lambda(\delta^3_{C_i}) = \{x \mapsto true\}$; and *iii*) for each $i \in [1..k]$, $\lambda(\delta^1_{C_i}) = ws_{i1} = \{x_j \mapsto true\}$, and $\lambda(\delta^2_{C_i}) = ws_{i2} = \{x_h \mapsto true\}$, where $C_i = x_j \vee x_h$.



P-$\text{Ext}^{sem}_{\text{IND-D}}(S)$ solvable in PTIME for $sem \in \{\mathtt{ad}, \mathtt{st}\}$

Definition

Given a prAAF $\mathcal{F} = \langle A, D, W, \lambda \rangle$ of the form IND-D and a set $S \subseteq A$ of arguments, the event that S is an admissible extension is $E_{\mathtt{ad}(S)} = e_1(S) \wedge e_2(S) \wedge e_3(S)$ where:

- $e_1(S) = \bigwedge_{a \in S} Lit(a)$
- $e_2(S) = \bigwedge_{\substack{\delta = (a,b) \in D \\ \wedge a \in S \wedge b \in S}} \neg Lit(\delta)$
- $e_3(S) = \bigwedge_{d \in A \setminus S} \left(e_{31}(S,d) \vee e_{32}(S,d) \vee e_{33}(S,d)\right)$ where:
  - $e_{31}(S,d) = \neg Lit(d)$
  - $e_{32}(S,d) = Lit(d) \wedge \bigwedge_{\substack{\delta = (d,b) \in D \\ \wedge b \in S}} \neg Lit(\delta)$
  - $e_{33}(S,d) = Lit(d) \wedge \bigvee_{\substack{\delta = (d,b) \in D \\ \wedge b \in S}} Lit(\delta) \wedge \bigvee_{\substack{\delta = (a,d) \in D \\ \wedge a \in S}} Lit(\delta)$

P-EXT$^{sem}_{\text{IND-D}}(S)$ solvable in PTIME for $sem \in \{\mathtt{ad}, \mathtt{st}\}$

(i) there exist $a, b \in S$ such that $Lit(a) = \neg Lit(b)$ hence $E_{\mathtt{ad}}(S) = false$;

(ii) it is possible to rewrite $E_{\mathtt{ad}}(S)$ into a boolean expression $\mathrm{REW}(E_{\mathtt{ad}}(S))$ equivalent to $E_{\mathtt{ad}}(S)$ having the following form:

$$\begin{array}{c} x_1 \wedge \cdots \wedge x_n \wedge \neg x_{n+1} \wedge \cdots \wedge \neg x_{n+m} \wedge \\ (E_1 \wedge \cdots \wedge E_k) \wedge \\ ((x_{n+m+1} \wedge E_{k+1}) \vee (\neg x_{n+m+1} \wedge E'_{k+1})) \wedge \ldots \\ \cdots \wedge ((x_{n+m+l} \wedge E_{k+l}) \vee (\neg x_{n+m+l} \wedge E'_{k+l})) \end{array} \tag{1}$$

where:

- for each $i, j \in [1..n+m+l]$, with $i \neq j$, we have $x_i \neq x_j$;
- for each $i \in [1..k+h+l]$, E_i (resp. E'_i) is a conjunction of boolean formulas, i.e., $E_i = E_{i1} \wedge \cdots \wedge E_{ih}$ (resp., $E'_i = E'_{i1} \wedge \cdots \wedge E'_{ih'}$), where every E_{ij} (resp. E'_{ij}) is a boolean formula of the form $E^*_{ij} \vee \neg E^*_{ij} \wedge E^\#_{ij}$. Herein, each E^*_{ij} and each $E^\#_{ij}$ are boolean formulas of the form $E^*_{ij} = \bigwedge_{i'=1}^{r} y_{i'} \wedge \bigwedge_{i'=r+1}^{r+r'} \neg y_{i'}$ and $E^\#_{ij} = \bigvee_{j'=1}^{s} z_{j'} \vee \bigvee_{j'=s+1}^{s+s'} \neg z_{j'}$, where each variable $y_{i'}$ (with $i' \in [1..r+r']$) and each variable $z_{j'}$ (with $j' \in [1..s+s']$) are distinct fresh variables having no other occurrences in the whole formula $\mathrm{REW}(E_{\mathtt{ad}}(S))$.

Bipolar Argumentation Frameworks (BAF)

- *Bipolar Abstract Argumentation Frameworks* (BAFs) allow *supports*, besides attacks, to be specified between arguments
  - A *bipolar abstract argumentation framework* (BAF) is a tuple $\mathcal{F} = \langle \mathcal{A}, \mathcal{R}_a, \mathcal{R}_s \rangle$, where $\mathcal{A}$ is a set of arguments, $\mathcal{R}_a \subseteq \mathcal{A} \times \mathcal{A}$ is a defeat/attack relation and $\mathcal{R}_s \subseteq \mathcal{A} \times \mathcal{A}$ is a support relation
- Two formal semantics of support considered in this talk
  - in [Cayrol and Lagasquie-Schiex 2005], the support is a generic "inverse" of the notion of attack ("*abstract semantics*")
  - in [Boella et Al. 2010], it is viewed as a "deductive" correlation between arguments ("*deductive semantics*")

Definition (supported attacks and d-attacks)

- There is a *supported attack* from an argument a to an argument b iff there is a sequence of supports from a to an argument a' and an attack from a' to b
- There is a *d-attack* from from an argument a to an argument b iff
  - a attacks b
  - there is an argument a' such that there is a path from a to a' consisting of only support edges, and a' d-attacks b, or
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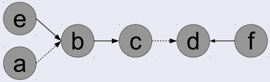

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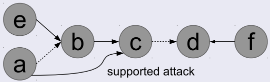

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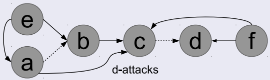

BAF semantics

The various extensions' semantics defined for AAFs have been shown to have a natural counterpart over BAFs, after noticing that combining attacks with supports (of any semantics) generates "implicit" attacks.

Definition (Conflict-free and safe sets of arguments)

A set of arguments $S \subseteq \mathcal{A}$ is:

- *conflict-free* iff $\nexists\, a, b \in S$ such that $\{a\}$ set-attacks b;
- *safe* iff $\nexists\, b \in \mathcal{A}$ such that S set-attacks b and either S set-supports b or $b \in S$.

Definition (Stable extension)

A set of arguments $S \subseteq \mathcal{A}$ is a stable extension iff S is conflict-free and $\forall a \in \mathcal{A} \setminus S$ it holds that S set-attacks a.

Definition (Admissible extension)

A set $S \subseteq \mathcal{A}$ is

- a *d-admissible* extension iff S is conflict-free and set-defends all of its arguments;
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BAF semantics

In turn, the other semantics subsuming the admissible one are defined as follows.

A set $S \subseteq \mathcal{A}$ is said to be:

– a *d-complete* (resp. *s-complete*, *c-complete*) extension iff S is d-admissible (resp., *s-admissible*, *c-admissible*) and S contains all the arguments set-defended by S;

– a *d-grounded* (resp. *s-grounded*, *c-grounded*) extension iff S is a minimal (w.r.t. $\subseteq$) d-complete (resp. *s-complete*, *c-complete*) extension;

– a *d-preferred* (resp. *s-preferred*, *c-preferred*) extension iff S is a maximal (w.r.t. $\subseteq$) d-complete (resp. *s-complete*, *c-complete*) extension;

– a *d-ideal* (resp. *s-ideal*, *c-ideal*) extension iff S is a maximal (w.r.t. $\subseteq$) d-admissible (resp. *s-admissible*, *c-admissible*) extension and S is contained in every d-preferred (resp. *s-preferred*, *c-preferred*) extension.

Probabilistic BAF

Arguments, attacks and supports can be uncertain (Constellation Approach)

Definition (Probabilistic BAF (PrBAF))

A *probabilistic BAF* (prBAF) $\mathcal{F}$ is a tuple $\mathcal{F} = \langle \mathcal{A}, \mathcal{R}_a, \mathcal{R}_s, \mathcal{P} \rangle$, where $\mathcal{F} = \langle \mathcal{A}, \mathcal{R}_a, \mathcal{R}_s \rangle$ is a BAF and $\mathcal{P}$ is a probability distribution function (pdf) over the set
$PS = \{\alpha = \langle \mathcal{A}', \mathcal{R}'_a, \mathcal{R}'_s \rangle \mid \mathcal{A}' \subseteq \mathcal{A} \wedge \mathcal{R}'_a \subseteq (\mathcal{A}' \times \mathcal{A}') \cap \mathcal{R}_a \wedge \mathcal{R}'_s \subseteq (\mathcal{A}' \times \mathcal{A}') \cap \mathcal{R}_s\}$.

- The elements in $PS(\mathcal{F})$ (*possible BAFs*) are the alternative cases of dispute that may occur, and each of them is encoded by a BAF

EX

A prBAF $\mathcal{F}$ of form EX is a tuple $\langle \mathcal{A}, \mathcal{R}_a, \mathcal{R}_s, \vec{\alpha}, \vec{\mathcal{P}} \rangle$, where

- $\vec{\alpha} = \alpha_1, \ldots, \alpha_m$ is the sequence of the possible BAFs that are assigned non-zero probability
- $\vec{\mathcal{P}} = P(\alpha_1), \ldots, P(\alpha_m)$ are their probabilities

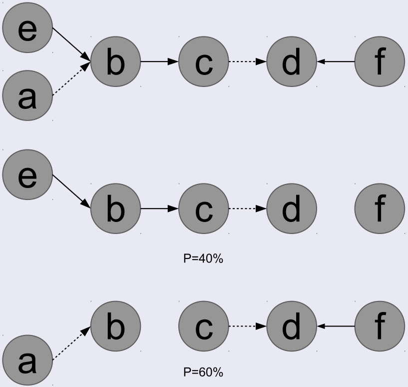
IND

A prBAF of type IND is a tuple $\langle \mathcal{A}, \mathcal{R}_a, \mathcal{R}_s, \mathcal{P}_\mathcal{A}, \mathcal{P}_\mathcal{R} \rangle$ where

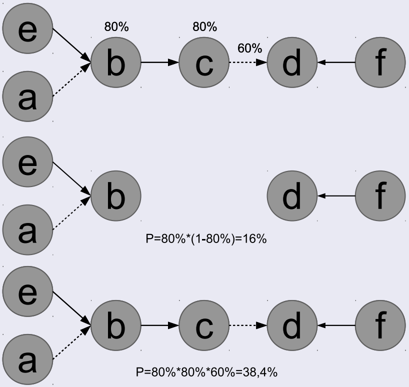
- $A = \{a_1, \ldots, a_m\}$, $\mathcal{R}_a = \{\delta_1, \ldots, \delta_n\}$
- $\mathcal{R}_s = \{\sigma_1, \ldots, \sigma_k\}$, and
- $\mathcal{P}_\mathcal{A} = \{P(a_1), \ldots, P(a_m)\}$,
- $\mathcal{P}_\mathcal{R} = \{P(\delta_1), \ldots, P(\delta_n), P(\sigma_1), \ldots, P(\sigma_k)\}$

Example of PrBAFs

Example PrBAF of type EX



Example PrBAF of type IND



Probability of extensions

- The probability $P^{sem}(S)$ that a set S of arguments is reasonable according to a given semantics *sem* is defined as *the sum of the probabilities of the possible worlds w for which S is reasonable according to sem*

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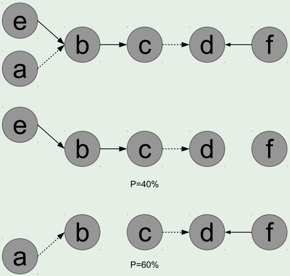
Example

The probability that $\{b\}$ is a d-/s-/c-admissible set is 60% since it is a d-/s-/c-admissible set only in the possible BAF on the bottom of the right hand side



Complexity of Probabilistic Abstract Argumentation

P-EXT$^{sem}(S)$ is the problem of *computing the probability* $P^{sem}(S)$

- P-EXT$^{sem}(S)$ is the probabilistic counterpart of the problem EXT$^{sem}(S)$ of verifying whether a set S is reasonable according to *sem*

	Literature			Our results		
sem	EXT (AAF)	P-EXT (prAAF) IND	P-EXT (prAAF) EX	EXT (BAF)	P-EXT (prBAF) IND	P-EXT (prBAF) EX
admissible	*P*	*FP*	*FP*	*P*		
stable	*P*	*FP*	*FP*	*P*		
complete	*P*	$FP^{\#P}$	*FP*	*P*		
grounded	*P*	$FP^{\#P}$	*FP*	*P*		
preferred	*coNP*	$FP^{\#P}$	$FP^{\|\|NP}$	*coNP*		
ideal	in θ_2^p, *coNP-h*	$FP^{\#P}$	$FP^{\|\|NP}$	in θ_2^p, *coNP-h*		

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The Main Result

Theorem

For any sem $\in$ SEM, and for both s- and d- prBAFs, $\text{P-EXT}^{sem}_{\text{IND}}(S)$ *is* $FP^{\#P}$*-complete.*

- Upper bound proved by defining a polynomial time algorithm $\mathcal{A}$ with access to a $\#P$ oracle that computes $P^{sem}(S)$
- Lower bound proved by showing a reduction from the $\#P$-hard problem $\#BP2DNF$ to $\text{P-EXT}^{sem}_{\text{IND}}(S)$ for s-PrBAF and from problem $\#P2CNF$ to $\text{P-EXT}^{sem}_{\text{IND}}(S)$ for d-PrBAF
  - $\#BP2DNF$ is the problem of counting the satisfying assignments of a bipartite positive 2-DNF formula.
  - $\#P2CNF$ is the problem of counting the number of satisfying assignments of a CNF formula where each clause consists of exactly 2 positive literals

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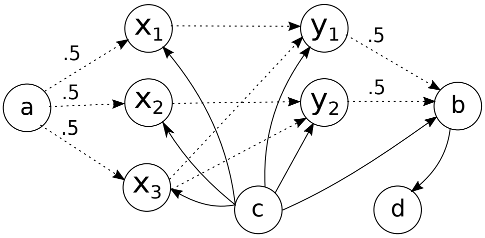
Proving the lower bound

Example (reduction for s-prBAF)

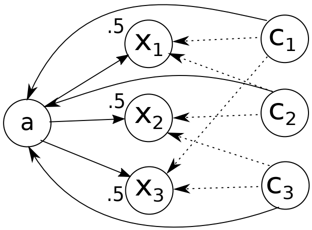
The s-prBAF representing the formula
$\phi_1 = (X_1 \wedge Y_1) \vee (X_2 \wedge Y_2) \vee (X_3 \wedge Y_1) \vee (X_3 \wedge Y_2)$

Example (reduction for d-prBAF)

The d-prBAF representing the formula
$\phi_2 = (X_1 \vee X_3) \wedge (X_1 \vee X_2) \wedge (X_2 \vee X_3)$



(a)



(b)

Incomplete AAFs

Definition ([Baumeister et al., 2018a])

An *incomplete Abstract Argumentation Framework* is a tuple $\langle A, A^?, D, D^? \rangle$, where:

- A and $A^?$ are disjoint sets of arguments,
- D and $D^?$ are disjoint sets of defeats between arguments in $A \cup A^?$

- The arguments in A are said to be *certain* (i.e., they are definitely known to exist), while those in $A^?$ *uncertain* (i.e., it is not known for sure if they occur in the argumentation or not).
- The defeats in D are said to be *certain* (i.e., they are definitely known to exist, if both the incident arguments exist), while those in $D^?$ *uncertain* (i..e, it is not for sure whether they hold in the argumentation, even if both the incident arguments exist).

(Remark)

What is their relationship with prAAFs?

Completions and extension verification problem

Definition (Completion)

A completion for an iAAF $I\!F = \langle A, A^?, D, D^? \rangle$ is an AAF $F = \langle A', D' \rangle$ where:

- $A \subseteq A' \subseteq (A \cup A^?)$
- $D \cap (A' \times A') \subseteq D' \subseteq (D \cup D^?) \cap (A' \times A')$

The notion of extension is re-formulated under both the possible and the necessary perspectiv

- true in *at least one* and *every* scenario, respectively.

An iAAF is similar to a prAAF of type ind, where certain arguments/defeats have probaility 1 and uncertain arguments/defeats have probaility p, with $0 < p < 1$.

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i-extension: definition and complexity of the verification problem

Definition (Possible and necessary i-extension)

Given an iAAF *IF* and a semantics σ, a set S is said to be a possible (resp., necessary) i-extension for *IF* (under σ) if, for at least one (resp., for every) completion $F = \langle A', D' \rangle$ of *IF*, the set $S^* = S \cap A'$ is an extension of F under σ.

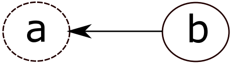
σ	i-extensions	
	pos	nec
`ad,st,co,gr`	*NP-c*	*P*
`pr`	Σ_2^p-c	*coNP-c*

From i-Extensions to i*-Extensions

(Remark)

As a matter of fact, under the possible perspective, the definition of i-extension proposed in [Baumeister et al., 2018a] raises some issues that can be hardly reconciled with the expectation that an extension should consist of a set of arguments that collectively represent a robust point of view in a dispute.

- A set S may be an i-extension of an iAAF IF even if, for every completion F where S's arguments occur all together, S is not an extension for F.
- As an undesirable consequence of the point above, a set S may be an i-extension of IF even if some of its arguments are definitely conflicting, due to certain attacks between them
- Even in a possible perspective, one would expect that the fact that an argument belongs to an i-extension (i.e., the argument is "i-acceptable") certifies that the argument is "robust to some extent". Unfortunately, an i-acceptable argument may not be robust at all.



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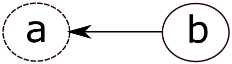

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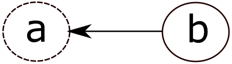

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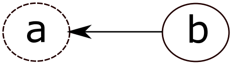

i*-extensions

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Proposition

Given an iAAF $IF = \langle A, A^?, D, D^? \rangle$ and a semantics $\sigma \in \{\mathtt{ad}, \mathtt{st}, \mathtt{co}, \mathtt{gr}, \mathtt{pr}\}$, let N^* and N be the sets of necessary i*- and i- extensions under σ, respectively, and P^* and P the sets of possible i*- and i- extensions under σ, respectively.

- $N^* \subseteq N \subseteq P^* \subseteq P$.
- Under both the possible and necessary perspectives, if S is an i-extension and $S \cap A = S$, then S is an i*-extension.
- The vice versa holds only for the necessary perspective.

Complexity overview

σ	i-extensions		i*-extensions	
	pos	nec	pos	nec
ad,st,co,gr	*NP-c*	*P*	*P*	*P*
pr	Σ_2^p-*c*	*coNP-c*	Σ_2^p-*c*	*coNP-c*

Conclusion an open problems

- We revised the complexity of P-EXT$^{sem}(S)$ and P-ACC$^{sem}(a)$ for prAAFs and prBAFs and introduced the framework GEN for compactly representing prAAfs
- Other issues not covered in this talk
  - [Liao et al., 2018] showed that P-EXT$^{sem}(S)$ is fixed-parameter tractable for the complete (and preferred) semantics for PrAAFs of type IND.
  - [Mantadelis and Bistarelli, 2020] defined the probabilistic attack/argument normal form for PrAAFs of type IND.

Conclusion an open problems

- Some open research problems
  - Identification of other tractable cases: are the results of [Liao et al., 2018] extendable to prAAFs of type IND-D?
  - What is the complexity of P-EXT$^{sem}(S)$ (P-ACC$^{sem}(a)$) under other semantics? For instance the complexity of P-EXT$^{sem}(S)$ have been characterized under **stage** semantics for prAAF of type ind, what about gen (and its restrictions)?
  - Normal forms proposed in [Mantadelis and Bistarelli, 2020] can be extended to prAAfs of type gen (or any of its restrictions)?
  - How to provide the pdf from data? [Hunter and Noor, 2020] presents an interesting approach for aggregating users' reviews in a pdf. It will be interesting to develop efficient and effective algorithms that use the proposed theoretical framework to derive a pdf compactly representing users' reviews.

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