An Introduction to Defeasible Logic

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5 September 2020

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Basic Defeasible Logic

Defeasibility: Reasonable results with minimum effort

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Factual omniscience and (non-)monotonic reasoning

$$PhD \rightarrow Uni$$

Defeasibility: Reasonable results with minimum effort

Factual omniscience and (non-)monotonic reasoning

$$PhD \rightarrow Uni$$
$$Weekend \rightarrow \neg Uni$$
$$PublicHoliday \rightarrow \neg Uni$$
$$Sick \rightarrow \neg Uni$$

Defeasibility: Reasonable results with minimum effort

Factual omniscience and (non-)monotonic reasoning

$$
\begin{aligned}
PhD &\rightarrow Uni \\
Weekend &\rightarrow \neg Uni \\
PublicHoliday &\rightarrow \neg Uni \\
Sick &\rightarrow \neg Uni \\
Weekend \wedge VICdeadline &\rightarrow Uni
\end{aligned}
$$

Defeasibility: Reasonable results with minimum effort

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$$Sick \rightarrow \neg Uni$$
$$Weekend \wedge VICdeadline \rightarrow Uni$$

VIC= Very Important Conference

Defeasibility: Reasonable results with minimum effort

Factual omniscience and (non-)monotonic reasoning

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Weekend \land VICdeadline &\rightarrow Uni \\
VICdeadline \land PartnerBirthday &\rightarrow \neg Uni
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Defeasibility: Reasonable results with minimum effort

Factual omniscience and (non-)monotonic reasoning

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$$PublicHoliday \rightarrow \neg Uni$$
$$Sick \rightarrow \neg Uni$$
$$Weekend \wedge VICdeadline \rightarrow Uni$$
$$VICdeadline \wedge PartnerBirthday \rightarrow \neg Uni$$

$$Phd \wedge (\neg Weekend \vee (Weekend \wedge VICdeadline \wedge \neg PartnerBirthday)) \wedge \neg Sick \ldots \rightarrow Uni$$

Inconsistent Information

- Classical logics “collapse” in the face of inconsistencies
  Everything can be derived
- But inconsistencies do happen in real settings
  Common when integrating knowledge from various Web sources
- Nonmonotonic reasoning is inconsistency tolerant reasoning

Defeasiblity: Example 1

TELECOMMUNICATIONS CONSUMER PROTECTIONS CODE (C628:2012)
Section 2.1. Definitions
Complaint means an expression of dissatisfaction made to a Supplier in relation to its Telecommunications Products or the complaints handling process itself, where a response or Resolution is explicitly or implicitly expected by the Consumer.

An initial call to a provider to request a service or information or to request support is not necessarily a Complaint. An initial call to report a fault or service difficulty is not a Complaint. However, if a Customer advises that they want this initial call treated as a Complaint, the Supplier will also treat this initial call as a Complaint.

If a Supplier is uncertain, a Supplier must ask a Customer if they wish to make a Complaint and must rely on the Customer's response.

Defeasiblity: Example 2

NATIONAL CONSUMER CREDIT PROTECTION ACT 2009 (Act No. 134 of 2009)
Section 29

(1) A person must not engage in a credit activity if the person does not hold a licence authorising the person to engage in the credit activity.

(3) For the purposes of subsections (1) and (2), it is a defence if:

  (a) the person engages in the credit activity on behalf of another person (the principal); and

  (b) the person is:

    (i) an employee or director of the principal or of a related body corporate of the principal; or

    (ii) a credit representative of the principal; and . . .

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Why Defeasible Deontic Logic

Rule-based non-monotonic formalism

- Flexible
- Efficient (linear complexity)
- Directly skeptic semantics
- Argumentation semantics
- Constrictive proof theory
- Encompasses other formalisms used in AI and Law
- Applied in several fields/optimised implementations
- Extensible
- Not affected by Deontic Logic Paradoxes

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Strength of Rules

Relationship between body (set of premises) and head (conclusion)

$$\textit{body} \times \textit{head}$$

body always head
body sometimes head
body not complement head
body no relationship head
body always complement head
body sometimes complement head
body not head

Defeasible Logic Blueprint

Simple sceptical rule based non-monotonic formalism.

- Derive (plausible) conclusions with the minimum amount of information.
  - Definite conclusions
  - Defeasible conclusions
- Defeasible Theory
  - Facts
  - Strict rules ($A_1, \dots, A_n \to B$)
  - Defeasible rules ($A_1, \dots, A_n \Rightarrow B$)
  - Defeaters ($A_1, \dots, A_n \leadsto B$)
  - Superiority relation over rules

Reasoning with Defeasible Logic

- Positive defeasible conclusions: meaning that the conclusions can be defeasible proved;
- Negative defeasible conclusions: meaning that one can show that the conclusion is not even defeasibly provable.

Conclusions in Defeasible Logic

A proof is a finite sequence $P = (P(1), \ldots, P(n))$ of tagged literals satisfying four conditions

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Proving Conclusions in Defeasible Logic

1. Give an argument for the conclusion you want to prove

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1. A is a fact; or
2. there is an applicable rule for A, and either
   1. all the rules for $\neg A$ are discarded (i.e., not applicable) or
   2. every applicable rule for $\neg A$ is weaker than an applicable rule for A.

. . . formally

$+\partial$: If $P(n+1) = +\partial q$ then

1) $+\Delta q \in P(1..n)$, or
2) $-\Delta {\sim} q \in P(1..n)$ and
    2.1) $\exists r \in R_{sd}[q]$: $\forall a \in A(r) + \partial a \in P(1..n)$ and
    2.2) $\forall s \in R[{\sim} q]$ either $\exists a \in A(s) : -\partial a \in P(1..n)$ or
        $\exists t \in R[q]$: $\forall a \in A(t) + \partial a \in P(1..n)$ and $t \succ s$.

Strong Negation

Negative Proof Tags are the strong negation of the corresponding positive ones:

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$-\partial$: If $P(n+1) = -\partial q$ then
    1) $-\Delta q \in P(1..n)$, and
    2) $+\Delta{\sim}q \in P(1..n)$ or
        2.1) $\forall r \in R_{sd}[q]$: $\exists a \in A(r) - \partial a \in P(1..n)$ or
        2.2) $\exists s \in R[{\sim}q]$: $\forall a \in A(s) : +\partial a \in P(1..n)$ and
            $\forall t \in R[q]$ such that $\exists a \in A(t) - \partial a \in P(1..n)$ or not $t \succ s$.

Example

Facts: A_1, A_2, B_1, B_2

Rules: $r_1: A_1 \Rightarrow C$

$r_2: A_2 \Rightarrow C$

$r_3: B_1 \Rightarrow \neg C$

$r_4: B_2 \Rightarrow \neg C$

$r_5: B_3 \Rightarrow \neg C$

Superiority relation:

$r_1 > r_3$

$r_2 > r_4$

$r_5 > r_1$

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Phase 1: Argument for C

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Phase 1: Argument for C
A_1 (Fact), $r_1 : A_1 \Rightarrow C$

Example

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Phase 2: Possible counterarguments

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Phase 3: Rebut the counterarguments

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Phase 2: Possible counterarguments

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$r_4: B_2 \Rightarrow \neg C$

$r_5: B_3 \Rightarrow \neg C$

Phase 3: Rebut the counterarguments

r_3 weaker than r_1

r_4 weaker than r_2

r_5 is not applicable

Modelling Exceptions in DL

$$tcpc_1: \mathit{ExpressionDissatisfaction} \Rightarrow \mathit{Complaint}$$
$$tcpc_2: \mathit{InformationCall} \Rightarrow \neg \mathit{Complaint}$$
$$tcpc_3: \mathit{ProblemCall}, \mathit{FirstCall} \leadsto \mathit{Complaint}$$
$$tcpc_4: \mathit{AdviseComplaint} \Rightarrow \mathit{Complaint}$$

where $tcpc_2 \prec tcpc_1$ and $tcpc_2 \prec tcpc_4$.

Some Facets of Defeasible Reasoning

Nonmonotonic Reasoning Options

- Sceptical vs Credulous
- Ambiguity Blocking vs Ambiguity Propagation
- Team Defeats vs No Team Defeat

Basic Reasoning

Suppose you have one pieces of evidence, Evidence A suggesting that the defendant is responsible.

Given: *EvidenceA* and the rule

$$EvidenceA \Rightarrow Responsible$$

Sceptical: *Responsible*
Credulous: *Responsible*

Conflict

Suppose that your legal system is based on presumption of innocence, and the somebody is guilty if responsibility is proved.

Given the rules

$$r_1 : \mathit{Responsible} \Rightarrow \mathit{Guilty}$$
$$r_2 : \Rightarrow \neg \mathit{Guilty}$$

Sceptical: $\neg \mathit{Guilty}$
Credulous: $\neg \mathit{Guilty}$
What about if we have $r_1 > r_2$ (same conclusions)

Sceptical vs Credulous

Suppose you have two pieces of evidence. Evidence A suggesting that the defendant is responsible, and Evidence A suggesting that the defendant is not responsible.

Given: *EvidenceA*, and *EvidenceB* and the rules

$$EvidenceA \Rightarrow Responsible$$
$$EvidenceB \Rightarrow \neg Responsible$$

Sceptical: no conclusions
Credulous: both conclusions

Sceptical vs Credulous and preference

Suppose you have two pieces of evidence. Evidence A suggesting that the defendant is responsible, and Evidence A suggesting that the defendant is not responsible. However, Evidence A is more reliable than Evidence B.

Given: *EvidenceA*, and *EvidenceB* and the rules

$$r_1 : EvidenceA \Rightarrow Responsible$$
$$r_2 : EvidenceB \Rightarrow \neg Responsible$$
$$r_1 > r_2$$

Sceptical: *Responsible*
Credulous: *Responsible*

Ambiguity Blocking vs Ambiguity Propagation

$$\begin{array}{lll} & \ldots & \Rightarrow q \\ \ldots & \Rightarrow p & \Rightarrow \neg q \\ \ldots & \Rightarrow \neg p & \end{array}$$

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$$\begin{array}{lll} & \ldots & \Rightarrow q \\ \ldots & \Rightarrow p & \Rightarrow \neg q \\ \ldots & \Rightarrow \neg p & \end{array}$$

Ambiguity blocking: q is not ambiguous

Ambiguity Blocking vs Ambiguity Propagation

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\begin{array}{llll}
 & \ldots & & \Rightarrow q \\
\ldots & \Rightarrow p & & \Rightarrow \neg q \\
\ldots & \Rightarrow \neg p & &
\end{array}
$$

Ambiguity blocking: q is not ambiguous
Ambiguity propagating: q is ambiguous

Ambiguity Propagation vs Ambiguity Blocking

Suppose you have two pieces of evidence. Evidence A suggesting that the defendant is responsible, and Evidence B suggesting that the defendant is not responsible. If the defendant is responsible, then he is guilty. and we have presupposition of innocence.

Given: $EvidenceA$, and $EvidenceB$ and the rules

$$
\begin{aligned}
EvidenceA &\Rightarrow Responsible \\
EvidenceB &\Rightarrow \neg Responsible \\
Responsible &\Rightarrow Guilty \\
&\Rightarrow \neg Guilty
\end{aligned}
$$

Ambiguity blocking concludes $\neg Guilty$
Ambiguity propagation does not concludes $\neg Guilty$ and fails to conclude $Guilty$.

Ambiguity Propagation vs Ambiguity Blocking

Suppose you have two pieces of evidence. Evidence A suggesting that the defendant is responsible, and Evidence B suggesting that the defendant is not responsible. If the defendant is responsible, then he is guilty. and we have presupposition of innocence. If the defendant was wrongly accused then he is entitled to compensation.

Given: *EvidenceA*, and *EvidenceB* and the rules

$$EvidenceA \Rightarrow Responsible$$
$$EvidenceB \Rightarrow \neg Responsible$$
$$Responsible \Rightarrow Guilty$$
$$\Rightarrow \neg Guilty$$
$$\neg Guilty \Rightarrow Innocent$$
$$Innocent \Rightarrow Compensation$$

Ambiguity blocking concludes *Compensation*
Ambiguity propagation does not conclude *Compensation*

Team Defeat vs No Team Defeat

$$r_1 : General \Rightarrow Attack$$
$$r_2 : Captain \Rightarrow \neg Attack$$
$$r_1 > r_2$$

$$r_3 : Bishop \Rightarrow Attack$$
$$r_4 : Priest \Rightarrow \neg Attack$$
$$r_3 > r_4$$

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Team Defeat concludes *Attack*
No Team Defeat does not conclude *Attack*

Weak and Strong Support

Suppose that a drunk person testified that the accused (not known to him) was in a location different from the crime scene at the time of the crime. Secure footage from high definition cameras shows the accused at the crime scene at the time of the crime.

$$r_1 : drunk \Rightarrow \neg CrimeScene$$
$$r_2 : camera \Rightarrow CrimeScene$$
$$r_3 : \neg CrimeScene \Rightarrow Alibi$$

$r_2 > r_1$

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Do we have scintilla of evidence to claim that the accuse was at the crime scene at the time of the crime?

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Is it reasonable to say that we have substantial evidence supporting for the same claim?

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Do we have scintilla of evidence to claim that the accuse was at the crime scene at the time of the crime?

Is it reasonable to say that we have substantial evidence supporting for the same claim?

Is it reasonable to claim that beyond any reasonable doubts the accused has an alibi?

More Conclusions in Defeasible Logic

A proof is a finite sequence $P = (P(1), \ldots, P(n))$ of tagged literals satisfying four conditions

- $+\Delta p$ ($-\Delta p$) p is (not) derivable monotonically.
- $+\partial p$ ($-\partial p$): p is (not) derivable using ambiguity blocking, team defeat

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- $+\delta p$: p is derivable using ambiguity propagation, team defeat

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- $+\delta p$: p is derivable using ambiguity propagation, team defeat
- $+\delta^* p$ p is derivable using ambiguity propagation, no team defeat
- $+\sigma p$: p is a credulous conclusion using team defeat

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- $+\delta p$: p is derivable using ambiguity propagation, team defeat
- $+\delta^* p$ p is derivable using ambiguity propagation, no team defeat
- $+\sigma p$: p is a credulous conclusion using team defeat
- $+\sigma^* p$: p is a credulous conclusion using no team defeat

More Conclusions in Defeasible Logic

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- $+\partial p$ $(-\partial p)$: p is (not) derivable using ambiguity blocking, team defeat
- $+\partial^* p$: p is derivable using ambiguity blocking, no team defeat
- $+\delta p$: p is derivable using ambiguity propagation, team defeat
- $+\delta^* p$ p is derivable using ambiguity propagation, no team defeat
- $+\sigma p$: p is a credulous conclusion using team defeat
- $+\sigma^* p$: p is a credulous conclusion using no team defeat
- $+\sigma^- p$: p is a credulous weak conclusion

Derivations in Defeasible Logic

1. Give an argument for the conclusion you want to prove
2. Consider all possible counterarguments to it
3. Rebut all counterarguments
   - Defeat the argument by a stronger one
   - Undercut the argument by showing that some of the premises do not hold

Derivations in Defeasible Logics: Ambiguity blocking

$+\partial p$

1) $\exists$ an applicable rule r pro p
2) $\forall$ rule t con p either:
   2.1) t is not applicable
   2.2) t is defeated by an applicable rule s pro p stronger than t

Derivations in Defeasible Logics: Ambiguity blocking

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$+\partial^* p$

1) $\exists$ an applicable rule r pro p
2) $\forall$ rule t con p either:
   2.1) t is not applicable
   2.2) t is defeated by r where r is stronger than t

Derivations in Defeasible Logics: Ambiguity propagation

$+\partial p$

1) $\exists$ an applicable rule r pro p
2) $\forall$ rule t con p either:
    2.1) t is not applicable
    2.2) t is defeated by an applicable rule s pro p stronger than t

Derivations in Defeasible Logics: Ambiguity propagation

$+\partial p$

1) $\exists$ an applicable rule r pro p
2) $\forall$ rule t con p either:
   2.1) t is not applicable
   2.2) t is defeated by an applicable rule s pro p stronger than t

$+\delta p$

1) $\exists$ an applicable rule r pro p
2) $\forall$ rule t con p either:
   2.1) t is not applicable
   2.2) t is defeated by a supported rule s pro p stronger than s

Ambiguity Blocking vs Propagation in DL

Ambiguity blocking

$+\partial$: If $P(i+1) = +\partial q$ then either
(1) $+\Delta q \in P(1..i)$ or
(2) (2.1) $\exists r \in R_{sd}[q] \forall a \in A(r) : +\partial a \in P(1..i)$ and
(2.2) $-\Delta{\sim}q \in P(1..i)$ and
(2.3) $\forall s \in R[{\sim}q]\ \exists a \in A(s) : -\partial a \in P(1..i)$

Ambiguity Blocking vs Propagation in DL

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(2) (2.1) $\exists r \in R_{sd}[q] \forall a \in A(r) : +\partial a \in P(1..i)$ and
(2.2) $-\Delta{\sim}q \in P(1..i)$ and
(2.3) $\forall s \in R[{\sim}q]\ \exists a \in A(s) : -\partial a \in P(1..i)$

Ambiguity propagating

$+\delta$: If $P(i+1) = +\delta q$ then either
(1) $+\Delta q \in P(1..i)$ or
(2) (2.1) $\exists r \in R_{sd}[q] \forall a \in A(r) : +\delta a \in P(1..i)$ and
(2.2) $-\Delta{\sim}q \in P(1..i)$ and
(2.3) $\forall s \in R[{\sim}q]\ \exists a \in A(s) : -\sigma a \in P(1..i)$

Derivations in Defeasible Logics: Support

$+\sigma p$

1) $\exists$ a supported rule r pro p
2) $\forall$ rule s con p either
    2.1) s is not applicable using ambiguity propagation (i.e., $-\delta, -\delta^*$)
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$+\sigma^- p$

$\exists$ a supported rule r pro p

Properties of Defeasible Logic

Theorem

Defeasible logic is consistent. $+d\ a$ *and* $+d\ \neg a$ *(for* $d \in \{\partial, \partial^*, \delta, \delta^*\}$*) cannot be both derived, unless they are already known as certain knowledge (facts)*

Theorem

Defeasible logic is coherent. $+\#a$ *and* $-\#a$ *cannot be derived from the same knowledge base.*

Theorem

Defeasible logic has linear complexity.

Argumentation Semantics

Argumentation

- Underlying logical language
- Definition of argument
- Definition of conflict between arguments
- Definition of the status of arguments

Acceptability Semantics

Let P be a defeasible theory. We define J_i^P (set of accepted arguments) as follows.

- $J_0^P = \emptyset$
- $J_{i+1}^P = \{a \in Args_P \mid a \text{ is acceptable w.r.t. } J_i^P\}$

The set of *justified arguments* in a defeasible theory P is $JArgs^P = \cup_{i=1}^{\infty} J_i^P$.

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Proof Trees

- If $b_1, \ldots, b_n$ label the children of h then there is a ground instance of a rule in R with body $b_1, \ldots, b_n$ and head h.
- If, in addition, h is not the root of the tree then the rule must be a strict or defeasible rule.

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If the rule at the root of a proof tree is strict or defeasible and the proof tree is finite we say it is a *supportive proof tree*.
If all the rules in a proof tree are strict then we say that it is a *strict proof tree*.

Arguments

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- An argument A is *strict* if every proof tree in A is strict.
- If an argument is not strict it is *defeasible*.
- An argument A for p is a *supportive argument* if the proof tree associated to A is supportive.

Arguments and Support

Proposition

Let P be a defeasible theory and p be a literal.

- $P \vdash +\Delta p$ *iff there is a strict supportive argument for* p.
- $P \vdash -\Delta p$ *iff there is no (finite or infinite) strict argument for* p

Proposition

Let P be a defeasible theory and p be a literal.

- $P \vdash +\sigma p$ *iff there is a supportive argument for* p.
- $P \vdash -\sigma p$ *iff there is no (finite or infinite) defeasible argument for* p

Conflicting Arguments

- An argument A *attacks* an argument B if a conclusion of A is the complement of a conclusion of B.

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- An argument A is supported by a set of arguments S if every proper subargument of A is in S.
- An argument A is *undercut* by a set of arguments S if S support an argument B attacking a proper subargument of A.

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Theorem

Let P be a defeasible theory and p be a literal.

- $P \vdash +\delta p$ *iff* p *is justified.*
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Theorem

Let P be a defeasible theory and p be a literal.

- $P \vdash +\partial p$ *iff p is justified.*
- $P \vdash -\partial p$ *iff p is rejected.*

Applications (and further readings)

Normative Reasoning

Tomorrow's lecture

Negotiation

- Guido Governatori et al. "A Formal Approach to Protocols and Strategies for (Legal) Negotiation". In: *Procedings of the 8th International Conference on Artificial Intelligence and Law*. Ed. by Henry Prakken. IAAIL. ACM Press, 2001, pp. 168–177. DOI: 10.1145/383535.383555
- Marlon Dumas et al. "A Formal Approach to Negotiating Agents Development". In: *Electronic Commerce Research and Applications* 2.1 (2002), pp. 193–207. DOI: 10.1016/S1567-4223(02)00016-9
- Thomas Skylogiannis et al. "DR-NEGOTIATE— A System for Automated Agent Negotiation with Defeasible Logic-Based Strategies". In: *Data & Knowledge Engineering* 63 (2007), pp. 362–380. DOI: 10.1016/j.datak.2007.03.004

Rational Agents

- Guido Governatori and Antonino Rotolo. "Defeasible Logic: Agency, Intention and Obligation. Deontic Logic in Computer Science". In: *7th International Workshop on Deontic Logic in Computer Science*. Ed. by Alessio Lomuscio and Donald Nute. LNAI 3065. Berlin: Springer, 2004, pp. 114–128
- Guido Governatori and Antonino Rotolo. "BIO Logical Agents: Norms, Beliefs, Intentions in Defeasible Logic". In: *Journal of Autonomous Agents and Multi Agent Systems* 17.1 (2008), pp. 36–69. DOI: 10.1007/s10458-008-9030-4
- Guido Governatori et al. "The Rational behind the Concept of Goal". In: *Theory and Practice of Logic Programming* 16.3 (2016), pp. 296–324. DOI: 10.1017/S1471068416000053
- Ho-Pun Lam and Guido Governatori. "Towards a Model of UAVs Navigation in Urban Canyon through Defeasible Logic". In: *Journal of Logic and Computation* 23.2 (2013), pp. 373–395. DOI: 10.1093/logcom/exr028

Argumentation

- Guido Governatori et al. “Argumentation Semantics for Defeasible Logics”. In: *PRICAI 2000: Topics in Artificial Intelligence*. Ed. by Riichiro Mizoguchi and John Slaney. LNAI 1886. Berlin: Springer-Verlag, 2000, pp. 27–37
- Guido Governatori et al. “Argumentation Semantics for Defeasible Logics”. In: *Journal of Logic and Computation* 14.5 (2004), pp. 675–702. DOI: 10.1093/logcom/14.5.675
- Guido Governatori et al. “Strategic Argumentation is NP-complete”. In: *21st European Conference on Artificail Intelligence* (Prague, 18–22 Aug. 2014). Ed. by Torsten Schaub, Gerhard Friedrich and Barry O'Sullivan. Amsterdam: IOS Press, 2014, pp. 399–404. DOI: 10.3233/978-1-61499-419-0-399

Dialogues

- Subhasis Thakur et al. "Dialogue Games in Defeasible Logic". In: *20th Australian Joint Conference on Artificial Intelligence* (Gold Coast, Australia, 2–6 Dec. 2007). Ed. by Mehmet A. Orgun and John Thornton. Vol. 4830. Lecture Notes in Artificial Intelligence. Heidelberg: Springer, 2007, pp. 497–506. DOI: 10.1007/978-3-540-76928-6_51
- Jenny Eriksson Lundström et al. "An Asymmetric Protocol for Argumentation Games in Defeasible Logic". In: *10 Pacific Rim International Workshop on Multi-Agents* (Bangkok, Thailand). Ed. by Aditya Ghose and Guido Governatori. Vol. 5044. LNAI. Heidelberg: Springer, 2008, pp. 219–231. DOI: 10.1007/978-3-642-01639-4_19
- Guido Governatori et al. "Dialogues on Moral Theories". In: *Deontic Logic in Normative Systems* (3–6 July 2018). Ed. by Ian Broersen et al. London: College Publications, 2018, pp. 139–155

Argumentation Systems

- Guido Governatori. "On the Relationship between Carneades and Defeasible Logic". In: *The 13th International Conference on Artificial Intelligence and Law* (Pittsburgh, PA, USA, 6–10 June 2011). Ed. by Kevin D. Ashley and Tom M. van Engers. ACM, 2011, pp. 31–40
- Ho-Pun Lam, Guido Governatori and Régis Riveret. "On ASPIC$^+$ and Defeasible Logic". In: *Proceedings of COMMA 2016* (Potsdam, Germany, 8 Sept.–12 Aug. 2016). Ed. by Pietro Baroni et al. Vol. 287. Frontiers in Artificial Intelligence and Applications. Amsterdam: IOS Press, 2016, pp. 359–370. DOI: 10.3233/978-1-61499-686-6-359

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- Grigoris Antoniou et al. "Representation Results for Defeasible Logic". In: *ACM Transactions on Computational Logic* 2.2 (2001), pp. 255–287. DOI: 10.1145/371316.371517
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